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SH044-0005: Ellipsoidal Surfaces for the Potential Field Source Surface Model

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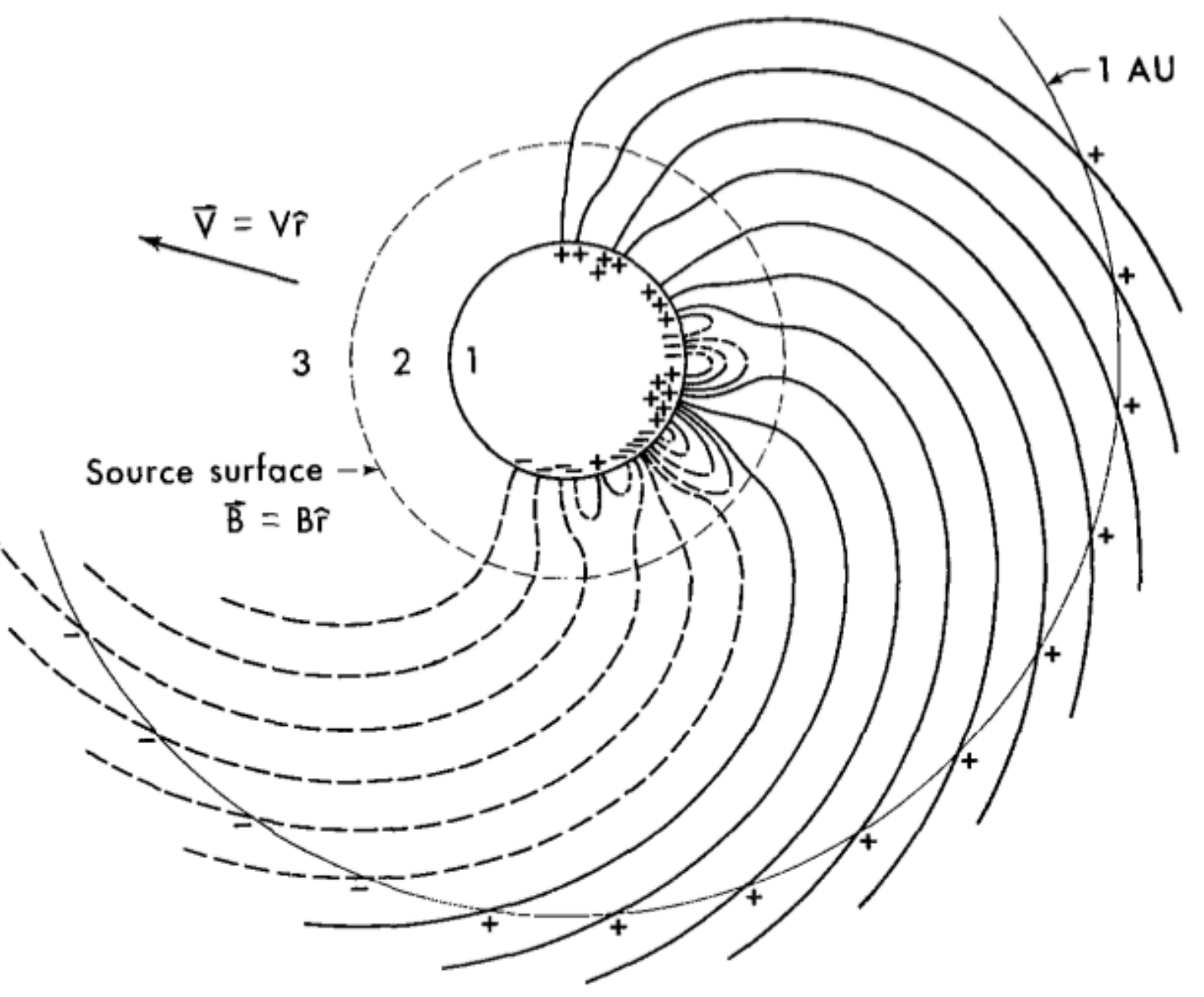
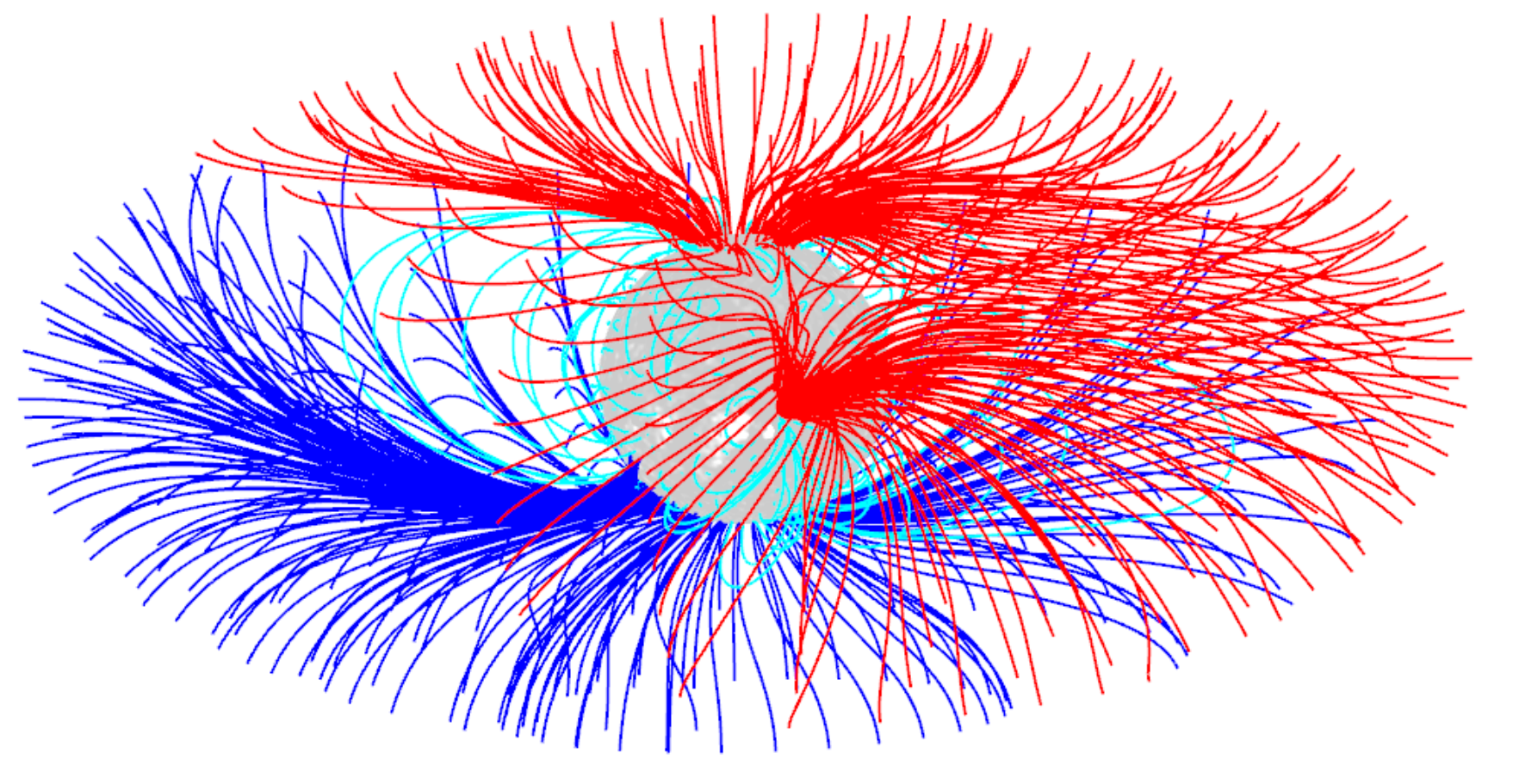
1 Introduction

The Potential Field Source Surface (PFSS) model [1, 2] computes a prediction of the solar corona's magnetic configuration. It assumes a magnetostatic, current-free computational domain between the photosphere and a virtual outer spherical boundary, the eponymous source surface.



Schematic of the PFSS model. The lower boundary (i.e., the photosphere) is the transition between regions 1 and 2. The computational domain of the model is region 2, bounded at the top by the source surface beyond which (region 3) the magnetic field is assumed to follow the Parker spiral. Image is taken from [2].

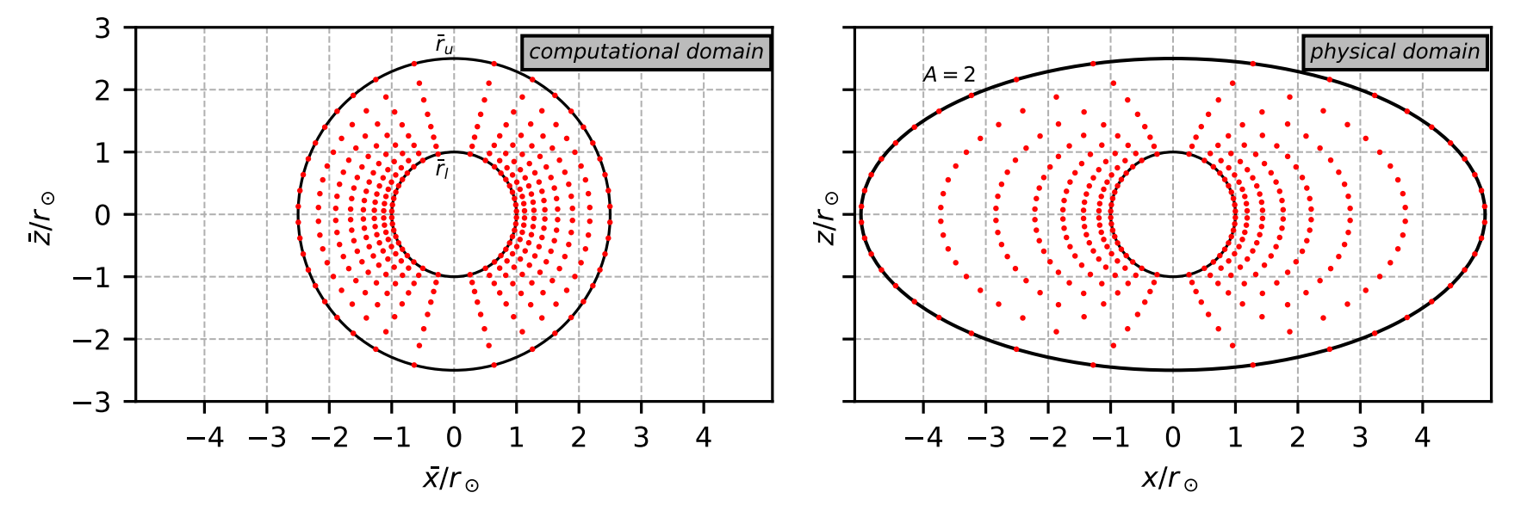
The source surface's spherical shape is just a first estimate that is readily incorporated into the model. Several authors have suggested that the PFSS model might benefit from relaxing this constraint and utilizing other shapes to act as the source surface [3, 4, 5, 6].
We incorporated an ellipsoid as the source surface into the PFSS model paradigm[7]. The resulting model predictions were then evaluated employing in-situ spacecraft data and a ballistic back mapping procedure.



PFSS solution with ellipsoidal source surface computed by our solver.

2 The improved PFSS model

First, a numeric PFSS implementation utilizing finite differences was implemented. This new solver was then checked to perform the same as a reference version from the Wilcox Solar Observatory (WSO). In a second step, this solver's mathematical framework was altered to feature either oblate or prolate ellipsoids as the source surface while retaining the possibility to employ a spherical source surface as a special case.



Example of the underlying computational (left) and physical (right) grids for a PFSS model with oblate ellipsoidal source surface. For visibility, the number of grid points is reduced. The stretching function $\bar{a}$ (see below) transforms the computational grid into the physical grid.

The resulting grid has spherical symmetry at the lower boundary (i.e., the photosphere) and ellipsoidal symmetry at the upper boundary (i.e., the source surface). A stretching function is utilized to achieve this configuration. A transformation between computational and physical coordinate systems allows an easier solution of the Laplace equation. Let computational coordinates be denoted by a bar, such as the cartesian coordinates $\bar{x}, \bar{y}$, and $\bar{z}$ and physical coordinates without a bar, such as x, y, and z. Let $\bar{r}$ be the radial coordinate in the computational domain, $\bar{r_u}$ and $\bar{r}_l$ be the upper and lower radial boundary heights and A the ellipticity. The stretching function $\bar{a}(\bar{r})$ to transform the computational to into the physical grid is then given by

$$\bar{a}(\bar{r}) = 1 + \frac{A-1}{\bar{r}_u^2 - \bar{r}_l^2}\left(\bar{r}^2 - \bar{r}_l^2\right) , \tag{1}$$

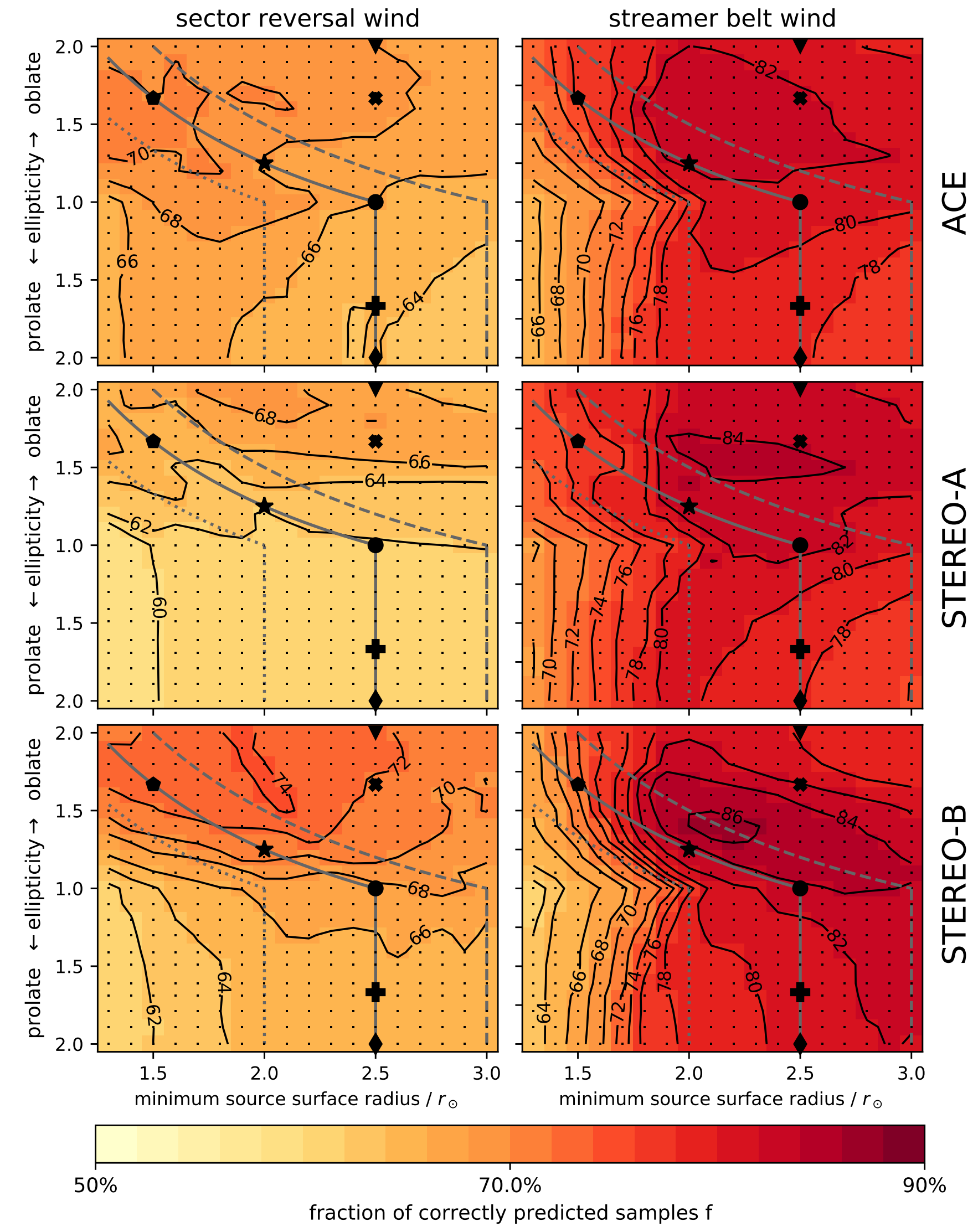
where the transformation of the cartesian coordinates (oblate case) is determined by

$$x = \bar{a}\bar{x} \ , \tag{2}$$
$$y = \bar{a}\bar{y} \ , \text{ and} \tag{3}$$
$$z = \bar{z} \ . \tag{4}$$

In the prolate case, the z-axis is stretched, and the x- and y-axes remain unchanged. As a result, the underlying curvilinear coordinate system is non-orthogonal in most locations.

3 Limitations

The improved PFSS model can now predict the coronal magnetic field configuration using a slight modification of the upper boundary condition. The source surface is still restricted to be a specific shape: To be spherical or ellipsoidal. Other shapes are not supported and require an extensive rework of the underlying mathematical solution process. The other limitations of the PFSS model still apply. The computational domain is assumed to be static and current-free for the entire duration of one Carrington rotation, thereby reducing the model's predictive power during solar activity maximum. Magnetic field lines are still considered to be oriented perpendicular to the source surface.



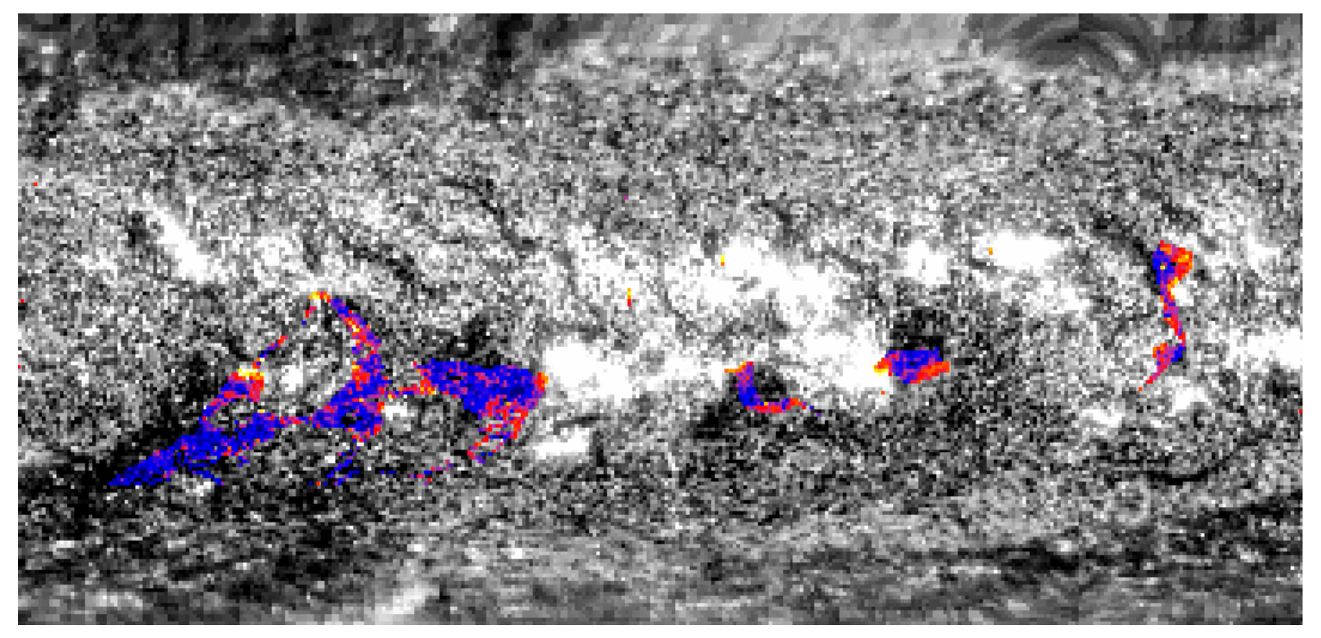
The fraction of correctly predicted samples for solar wind samples classified as either sector reversal (left column) or streamer belt (right column) wind. The back mapping polarity measure is performed for the spacecraft ACE (first row), STEREO-A (second row), and STEREO-B (third row) during solar activity minimum in 2006 (Carrington rotations 2066-2075). Magnetograms from the Michelson-Doppler-Imager (MDI) aboard the Solar and Heliospheric Observatory (SOHO) were utilized. The x-axes vary the minimum source surface height. The y-axes vary the ellipticity of the source surface. $y = 1$ depicts a spherical source surface. Moving up on the y-axis increases oblate ellipticity while moving down increases prolate ellipticity. Image is taken from [8].

4 Evaluation

We developed a measure to check whether this altered PFSS model is superior to the classical PFSS model with a spherical source surface [8]. The procedure employs in-situ spacecraft data from the Advanced Composition Explorer (ACE) and the twin Solar and Terrestrial Relations Observatories (STEREO) and is named the *back mapping polarity measure*. It classifies the measured plasma samples according to the Xu-Borovsky scheme [9], performs a ballistic back mapping along the Parker spiral to the source surface and checks, whether the predicted magnetic polarity at the source surface matches the magnetic polarity measured in-situ. The procedure is conducted for a wide variety of source surface ellipticities and heights, and the correctly predicted sample fraction is plotted to illustrate the performance of the new PFSS implementation.
As can be seen, the model performs slightly better for oblate ellipticities of the source surface. However, due to the spacecraft utilized for this study positioned in the ecliptic, higher latitudes are not included by the back mapping polarity measure. Prolate source surfaces facilitate changes to the model predictions, mostly at higher latitudes. Therefore, the analysis process does not do justice to these specific shapes, and additional evaluation procedures must be employed.

5 Outlook

The back mapping polarity measure is a simple tool for evaluating the source surface's magnetic field predictions. The configuration below the source surface is not considered. Due to the spacecraft measuring in the ecliptic, higher latitudes are not sampled. We propose to remedy these shortcomings by performing additional analyses using Extreme Ultra-Violet (EUV) synoptic maps of the photosphere. Open magnetic field lines (i.e., field lines originating on the photosphere and reaching the source surface) allow plasma parcels to leave the corona, whereas closed field lines act as magnetic traps. Therefore, photospheric footpoints of open field lines should map to darker regions in the EUV maps compared to closed field line footpoints. More accurate model parameters can be identified by varying source surface ellipticity and height and examining the average brightness of field line footpoints.



Footprints of field lines tracked from source surface to photosphere (colored) on top of SOHO EIT 304$\mathring{A}$ map (greyscale).

The model can be improved further in several ways. For example, the ellipsoidal source surface could be included with other PFSS paradigm improvements, such as the Current Sheet Source Surface (CSSS) model [10, 11]. Another improvement can be obtained by incorporating a fully numerical grid generation technique, thereby permitting almost arbitrary shapes to act as source surfaces. Although this would allow for the most realistic source surfaces, finding the exact shape to be utilized is another problem that needs to be addressed.

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